Interest Elasticity in a Life Cycle Model with Precautionary Savings.

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Simulations of life cycle models of wealth accumulation are often used to study the effect of policies aimed at encouraging savings. Examples are a reduction of taxes on capital, and tax favored forms of savings such as IRAs. These policies may have different characteristics and different goals (in the previous cases, to increase the total productive capacity of the country, and to encourage adequate asset accumulation for retirement purposes), but they share one common mechanism: they raise the after tax rate of return on savings. To study their effectiveness, it is therefore vital to understand if, and by how much, households increase their savings in response to more attractive interest rates.

There is a large literature on the interest elasticity, both theoretical and empirical. In this paper, I compute the interest elasticity of wealth implied by a life cycle model with precautionary savings. Summers (1981) suggested that, for reasonable calibrations, life cycle models imply very large interest elasticities of saving, although Evans (1983) pointed out that some parameter configurations may deliver much lower values. These models however assume certainty: wealth is only accumulated to provide consumption after retirement (and possibly to leave a bequest). Uncertainty and precautionary savings are ignored. In contrast, a precautionary saving model with infinitely lived agents, such as the one studied by Carroll (1992), implies that households simply accumulate a buffer stock of wealth to be used against negative shocks, but do not increase their wealth after they have reached such a level, and the saving rate can be on average zero. As shown in Carroll (1992), the interest rate has little impact on wealth accumulation in such a model.

This paper examines the question in a life cycle model with precautionary savings, where both motives for saving (retirement and precautionary) are present. While uncertainty may be the most important motive for younger households, eventually households will start saving for retirement as well. I will compute by how much the wealth of the median household at retirement changes after an exogenous permanent increase in the after tax interest rate.[1] I find that, in an estimated life cycle model with precautionary savings, the elasticity of savings is also very low (as in the pure precautionary model). The results are however sensitive to the utility parameters used.

I. The Model

I consider a life cycle model of consumption and savings with exogenous and stochastic earnings, uncertain lifespan, and possible receipt of bequests; the exact formulation can be found in Cagetti (2000). I assume a constant relative risk aversion utility function, $U(C_t) = C_t^{1-\gamma}/(1-\gamma)$, discounted by a factor β^t. Unfortunately, the CRRA formulation ties together the coefficient of risk aversion γ and the elasticity of intertemporal substitution $1/\gamma$; however, because of its simplicity, it is the most commonly used utility function in life cycle simulations.

Also for simplicity, there is only one type of asset, which is riskless and has a constant after tax rate of return r (3% in the baseline case). The main source of uncertainty are earnings. As standard in other papers studying precautionary savings, I assume that log after tax earnings have a deterministic component (function of age and education) and a unit-root stochastic component. The variability of the latter depends on education; the estimates are taken from Carroll et al. (1997).

It is worth pointing out that, in this setup, there are no bequest motives, and all bequests are accidental. In Cagetti (2000) I show that, given the parameter estimates, bequest motives are not relevant to determine wealth accumulation for the median household before retirement, and they are thus ignored in this paper.

II. The interest elasticity of wealth

The life cycle model can be solved numerically and simulated for various levels of the interest rate, to study by how much the median wealth increases, if the household can invest at a higher interest rate throughout his entire life cycle . To measure this effect, I report the interest elasticity of the median wealth at retirement, defined as $\frac{\partial W^m}{\partial r}\frac{r}{W^m}$, where r is the after tax interest rate (.03 in the baseline case), and W^m is the median wealth level for households near retirement (age 60-65), which is the variable of interest for some of the policy questions on the adequacy of retirement savings.

First, I compute the elasticity for the parameters values estimated in Cagetti (2000), where I use an after tax real interest rate of 3% and estimate a coefficient of risk aversion γ and a discount factor β, respectively, of 3.23 and .971 for college graduates, 3.93 and .883 for high school graduates, and .93 and 2.38 for households without a high school degree. The estimates are obtained by matching the empirical and the simulated age profile[2] of median wealth holdings over the life cycle, using data on total net worth, from a sample of households from the Panel Study of Income Dynamics restricted to households of age 25 to 65 composed at least by a head and a spouse.[3] These parameter values suggest that the precautionary motive is a quantitatively relevant determinant of savings; in the absence of uncertainty, households would start saving for retirement only after age 45-50, and would hold an amount of wealth at retirement from 30% (for college graduates) to 60% lower (for the other groups).

The resulting interest elasticities are .19 for college graduates, .10 for high school graduates, and .48 for people without a high school degree. These values are extremely small. For instance, the value for college graduates means that an increase in the interest rate from 3% to 4% (an increase by one third) would increase the total wealth held at retirement by approximately $.19 \cdot .33$, or only 6.2%. Given that the simulated median wealth (in 1992 dollars) is approximately 465,000 dollars, this would correspond to an increase of around 28,000 dollars. For the other groups, the increase would be 5,500 dollars for high school graduates (from 167,000), and 11,000 dollars for high school dropouts (from 72,000).

A 33% increase in the after tax interest rate over the total wealth would be quite large relative to most of the saving incentives studied. For instance, IRAs defer taxes only on some specific assets, which constitute only a fraction of the total wealth. Yet even this change would have only a small increase in total wealth. This result confirms, for instance, the finding of Engen et al. (1994), who also use parameter values similar to those estimated here, that IRAs did not significantly increase savings.

The small reactivity of savings can be explained by the fact that in the pure precautionary saving model, household want to build up a buffer stock of wealth, but will not save more once such level is reached, and thus will not react much to better investment opportunities.

III. Other parameter configurations.

Table 1 shows the value of the interest elasticity under different configurations of β and γ. In this exercise, I have changed the coefficient of risk aversion to 1, 3, and 5, and reestimated β alone. In other words, I have changed γ while at the same time keeping approximately the same median wealth at retirement.[4] The case of log utility implies a much lower intertemporal elasticity of substitution, and at the same time a lower importance of precautionary savings. Correspondingly, the elasticity is much higher, and for all groups it is greater than 1. The opposite occurs for the case of $\gamma = 5$. Clearly, the elasticity varies with the parameter chosen, as shown in the certainty case by Evans (1983).

Since the same parameter regulates risk aversion and intertemporal substitution, it is impossible to disentangle the effects of precautionary savings from that of pure intertemporal substitution. To gain some insight, I compute the life cycle profiles of wealth accumulation for the previous model, shutting down all the sources of uncertainty (in earnings, life span and bequests). I keep the coefficient of risk aversion at the estimated value, and as before I adjust the discount factor in order to match the same level of median wealth at retirement. The resulting elasticities are .26 for college graduates, .21 for high school graduates, and .98 for high school dropouts. The elasticities do indeed increase from the case with uncertainty. However, they do not increase as much as when moving, for instance, from $\gamma = 3$ to log utility. This suggests that both forces have an impact on the elasticities, and it may be useful to try to disentangle the two when studying the effects of changes in interest rates. Parametrizations of the utility function that distinguish the two (such as various non time additive recursive preferences) are however much more difficult to compute and simulate numerically.

IV Conclusions.

Life cycle models of wealth accumulation in which the precautionary motive is quantitatively relevant imply also extremely low interest elasticities, in contrast to the suggestions of many parametrizations of life cycle models without uncertainty (e.g. Summers 1981). Therefore, simulations of these models to study various policies (such as tax favored forms of savings) that raise the returns to savings will tend to show small effects on the wealth accumulation of households. These schemes should not be justified on the basis of the increased returns to savings in a standard time additive utility model. Other mechanisms, however, may provide a rationale. For instance, Laibson et al. (1998) focus on the irreversibility of some tax preferred forms of savings, which may generate large increases in savings in a model with hyperbolic discounting. The mechanism generating the results is, however, the irreversibility together with hyperbolic discounting, not the interest rate.

However, even in the standard model, the results are sensitive to different assumptions on the coefficient of risk aversion and on the discount factor. Lower parameters of risk aversion (close to log utility) generate much larger effects. Since there is still a debate regarding their values, the choice of the parameters used in the simulations can be very important.

The model presented can say much less about questions related to the total aggregate wealth of the economy. The aggregate wealth implied by the model is approximately one half of the total wealth in the U.S. economy. The model underestimates dramatically the amount of wealth held by the richest 5% of the population, a fraction that, according to the Survey of Consumer Finances, holds approximately 50% of the total net worth. A model in which the precautionary motive is very important for most of the population is consistent with very high elasticities of aggregate wealth at the macroeconomic level, if in fact the top 5% of the wealth distribution behaves differently than the rest of the population. For instance, these households may be more patient and have lower risk aversion, and be motivated by large and relevant bequest motives, which would generate higher elasticities. Many of these rich households are entrepreneurs, whose business activity may be affected by taxes. In the aggregate, therefore, one may still find large effects, for instance, of reducing the tax on capital. However, microeconomic justifications for such effects should be looked for in the behavior of these groups, rather than in the rest of the population.

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Footnotes.

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1. It is only a partial equilibrium exercise, but, as explained later, the median wealth is much lower than the average wealth, and therefore this life cycle model cannot explain the aggregate wealth.

2. Gourinchas and Parker (2000), who consider mean consumption profiles, were the first to estimate preference parameters from simulated life cycle models.

3. Singles are excluded because I ignore the disincentives to various social programs. They can be an important determinant of the behavior of these households, as argued by Hubbard et al. (1995).

4. The implied β are, respectively for γ=1,3,5, .927, .974, .990 for college graduates, .985, .930 and .824 for high school graduates, .973, .903 and .779 for high school dropouts.

Table 1

	Risk aversion		
	1	3	5
College	1.31	.26	.10
High school	1.18	.19	.07
No high school	1.96	.31	.13